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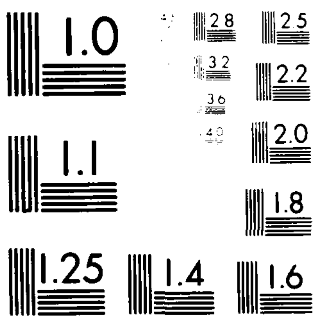

MICROCOPY RESOLUTION TEST CHART

LEVEL II (12) USAFA-TR-9

FAILURE-FREE RELIABILITY TESTS

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SEPTEMBER 1981

FINAL REPORT

DEAN OF THE FACULTY

UNITED STATES AIR FORCE ACADEMY

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A failure-free reliability test is compared with a fixed-time test under a two-parameter Weibull failure distribution. The distribution of the length of the failure-free test is derived, along with conditional failure probabilities for items still operating at the end of the original burn-in time. Graphs are provided for calculating expected test length and conditional failure probabilities.

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FAILURE-FREE RELIABILITY TESTS

by

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September 1981

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1. Introduction

In this report we analyze the probability distribution of the length of a failure-free reliability test and the conditional reliability $r(t, \Delta t)$ of components undergoing the reliability test. The conditional reliability of a component we define as the *probability of a component* not failing for Δt time units in the future, given that the component is operating at time t,

$$r(t, \Delta t) = P[\, T > t + \Delta t \mid T > t \,] \tag{1}$$

We assume a two-parameter Weibull life-length distribution with scale parameter δ and shape parameter β. When $\beta = 1$, the Weibull coincides with the exponential distribution, so that (1) reduces to the reliability at time Δt,

$$r(t, \Delta t) = R(\Delta t) = P[\, T > \Delta t \,] \tag{2}$$

Although the results contained herein are valid for any values of $\beta > 0$, we restrict ourselves to a reliability growth situation in which $\beta < 1$.

We use these ideas to compute the expected length of a failure-free test and contrast it with a fixed-length test whose length is equal to the expected length of the failure-free test. We can then examine the conditional reliability of a component at the end of the original burn-in time. The results of this report can be used by the test engineer to answer the following questions:

(a) How much longer than the original burn-in time can a failure-free test be expected to run?

(b) How many more failures can be expected in a fixed-time test with the same length as the failure-free test?

Based on the answers to these questions, the test engineer can decide which reliability test, fixed-length or failure-free, would be more useful. For ease of application, we provide some charts which can be easily used to determine the length and conditional reliability of a failure-free test.

In section 2 of this report we describe in detail a failure-free reliability test. In order to perform the analysis, we must make a number of assumptions which we explicitly state in section 3. In section 4 we derive the equation of the probability distribution of the test length, and in section 5 we derive the conditional reliability equation. In section 6 we discuss briefly the numerical solutions of these equations and the estimation of parameters. In section 7 we describe the testing procedure and provide some numerical examples. Appendix A contains charts for the test length, and Appendix B contains charts for the conditional reliability.

2. Failure-Free Reliability Tests

A failure-free reliability test is one in which a system must pass a fixed time, t_0, free of failure before being accepted. If a failure is encountered before time t_0, the system is repaired and a new burn-in period of length t_0 is re-established. This reliability testing philosophy has merit in those situations in which there is suspected non-homogeneity of quality among various systems. In those situations, those systems that are 'likely to fail' will receive a great deal of testing before being accepted. On the other hand, other systems will be accepted with the original burn-in time t_0.

This type of reliability testing procedure may not be optimal in certain other situations. For instance, it may be difficult to justify an a priori assumption of non-homogeneity of quality among systems. In fact, the systems may all have identical time-to-failure probability distributions which are unchanged following repair. However, due to strictly random failure times, some components may appear to be 'likely to fail' and thus receive an inordinate amount of burn-in. Other components, on the other hand, may be accepted after time t_0 although they would have failed shortly after.

A simple example of the above is a series system made up of n devices. Suppose that the times to failure of all of the devices are independent, identically distributed (iid), with an exponential life-length distribution

$$f_i\,(t) = \lambda e^{-\lambda t},\ i = 1,\ 2,\ \ldots,\ n \tag{3}$$

Since this series system will fail when the first device fails, it follows that the system's life-length distribution is that of the smallest order statistic, which is

$$f(t) = n\,\lambda\ e^{-n\lambda t}, \tag{4}$$

again exponential. Now, if the system fails and the inoperative device is replaced by one with an identical life-length distribution, then the memoryless property of the exponential guarantees that the system will have the same life-length distribution (1). It is erroneous, then to conclude that this system requires a longer burn-in period than the others, since in fact they all have identical life-length distributions. If the device life-length distribution is Weibull, then the smallest order statistic still has a Weibull distribution. In this case, however, the reasoning is not as direct, since the other components have 'aged' and therefore have a different life-length distribution from that of the repaired system. In this report we examine this situation, subject to certain background assumptions.

3. <u>Problem Formulation</u>

The model which we consider is based on the following assumptions:

(i) The life-lengths of the system are iid following a two-parameter Weibull distribution:

$$f(t) = \beta/\delta\,(t/\delta)^{\beta-1} \exp[-(t/\delta)^{\beta}],\ t > 0,\ \delta > 0,\ 0 < \beta \leq 1 \quad (5)$$

The restriction that $\beta \leq 1$ is not strictly necessary. For the purpose of this report, however, we restrict ourselves to this case, which represents a reliability growth situation (or: decreasing hazard rate). The parameter δ is referred to as the characteristic life, in that 63.2% of the components on the average will fail before δ, independent of the value of β. In later sections, we will re-parametrize the terms of

$$\gamma = t/\delta \quad (6)$$

and thus interpret γ as a fraction of the characteristic life.

The Mean Time Between Failure (MTBF) for such a system is given by

$$\text{MTBF} = \delta\,\Gamma\,(1 + 1/\beta) \quad (7)$$

The reliability function, $R(t)$, is given by

$$R(t) = \exp[-(t/\delta)]^{\beta} \quad (8)$$

If $\beta = 1$, this distribution reduces to an exponential with MTBF $= \delta$.

(ii) All of the systems are simultaneously burned in for t_0 time units. If a system fails before time t_0, it is removed from test, repaired, and its post-repair condition is the same as a new system. That is, its life-length distribution is as given in (5). We further assume that there is independence between the life-lengths before and after repair. This would not be the case, for example, if there were a common source of failure which went undiagnosed. Assuming that the true cause of failure were diagnosed and corrected each time, however, independence of life lengths is not an unreasonable assumption. Since it takes some time to effect the repair, a new burn-in cycle for the repaired system is begun at the end of the original cycle; that is at time t_0. All repaired systems are then burned in from time t_0 until time $2t_0$, with the process being repeated for those systems which fail once again. A system is not accepted until it passes a time period of length t_0 which is failure-free.

4. Length of a Failure-Free Test (one system)

The length, L of a failure-free test is a random variable, and under the assumptions above,

$P[L = kt_0] = P$ [Failure during first k-1 cycles and success during the k^{th} cycle]

By independence it follows that

$$P[L = kt_0] = [1 - R(t)]^{k-1} R(t), \; k = 1, 2, 3, \ldots \tag{9}$$

which, using (8) becomes

$$P[L = kt_0] = \{1 - \exp[-(t_0/\delta)^{\beta}]\}^{k-1} \exp[-(t_0/\delta)^{\beta}] \tag{10}$$

If we define the random variable

$$W = L/t_0 \tag{11}$$

(where W = number of burn-in periods required to 'failure free')

then W follows a geometric distribution.

From this observation it follows that

$$E(W) = \exp (t_0/\delta)^{\beta} \tag{12}$$

and

$$\mathrm{Var}(W) = \exp[2(t_0/\delta)^{\beta}] - \exp(t_0/\delta)^{\beta} \tag{13}$$

Using (11), therefore, we obtain

$$E(L) = t_0 \exp (t_0/\delta)^{\beta} \tag{14}$$

and

$$\mathrm{Var}(L) = t_0^2 \{\exp[2(t_0/\delta)^{\beta}] - \exp(t_0/\delta)^{\beta}\}$$

Of more usefulness than expected value of test length, E(L), is the expected proportional excess test time over t_0, which we define as

$$L_x = (E(L) - t_0)/t_0 \tag{15}$$

Using (11), we obtain

$$L_x = \exp (t_0/\delta)^{\beta} - 1 \tag{16}$$

5. Probability of Failure

The time interval

$$I = (t_0, E(L)) \tag{17}$$

represents the expected additional time that a failure-free test extends over a fixed-time test of length t_0. We now consider the system reliability over the time interval I. Specifically, we derive the probability of system failure over I. If this probability is sufficiently high, then rather than use a failure-free procedure, the test engineer would want to extend the test to time E(L) equally for all systems. In either case, the expected total test time devoted to all systems is the same, namely

$$\text{Total Test Time} = n\, E(L) \tag{18}$$

Therefore, the test engineer can make a decision at time t_0 between two approaches which have approximately the same cost (since the total test times are equal, and assuming the test cost is proportional to its length), but one of which will have a higher pay-off in terms of defects found.

We are interested in

$$h(t_0;\, \delta, \beta) = P\,[\text{Failure in } I \mid T > t_0] \tag{19}$$

Such a conditional probability can be interpreted as the fraction of those systems still operating at time t_0 which will fail during time interval I. The conditional reliability will then be given by

$$r(t_0, E(L) - t_0) = 1 - h\,(t_0;\, \delta, \beta). \tag{20}$$

Since $h(t_0;\, \delta, \beta) = P\,[\text{Failure in } I]\,/P\,[T > t_0]$ (21)

$$= [R(t_0) - R[E(L)]]\,/R(t_0) \tag{22}$$

$$= 1 - R\,[E(L)]\,/R(t_0) \tag{23}$$

we have

$$h(t_0;\, \delta, \beta) = 1 - \exp\,\{(t_0/\delta)^\beta\,[1-\exp\,(\beta\,(t_0/\delta)^\beta)]\} \tag{24}$$

and

$$r(t_0, E(L) - t_0) = \exp\{(t_0/\delta)^\beta [1 - \exp(\beta\gamma^\beta)]\} \tag{25}$$

6. Expected Repeat Failures

The expected additional number of repeat failures for a failure-free test, RFF, may be computed using the developed distributions as

$$RFF = n[\exp(t_0/\delta)^\beta - 1] - n[1 - \exp - (t_0/\delta)^\beta]. \tag{26}$$

These two terms may be interpreted as the total expected number of failures and the expected number of failures in the first burn-in period, respectively. If a fixed-time test is used, and failed items are replaced, then the expected additional number of repeat failures, RFT, may be computed by

$$RFT = r[1 - \exp\{-(L_x t_0/\delta)^\beta\}] \tag{27}$$

where r = number of failures in time t_0.

6. Numerical Methods

Included with this report is a set of charts (Appendices A, B) which contain solutions of equations 16 and 24 for L_x and h as a function of the parameters β and $\gamma = t/\delta$. These charts were produced using a Newton-Raphson algorithm, solving iteratively for γ in the equations

$$\gamma_n = \gamma_{n-1} - \frac{h(\gamma\ ,\beta) - h^*}{\frac{\partial}{\partial \gamma}[\, h(t_0;\, \delta,\, \beta)]} \tag{28}$$

$$\gamma_n = \gamma_{n-1} - \frac{L_x(\cdot\,,\, \beta) - L_x^*}{\frac{\partial}{\partial \gamma}[\, L_x\ (\gamma,\, \beta)]} \tag{29}$$

where h^* and L_x^* are constants. Figures 1 through 5 are solutions of h and Figures 6 through 9 are solutions of L_x. The figures differ in their range of values, as listed in Tables I and II. The computations were performed on the Air Force Academy's Burroughs B6700 computer.

Table I. Ranges for $h(\gamma, \beta)$ graphs

Figure	β	γ	h
1	0.0 - 1.0	0.00 - 0.01	0.01 - 0.10
2	0.0 - 1.0	0.00 - 0.01	0.10 - 1.00
3	0.0 - 1.0	0.01 - 0.10	0.02 - 0.10
4	0.0 - 1.0	0.01 - 0.10	0.10 - 1.00
5	0.0 - 1.0	0.10 - 1.00	0.20 - 1.00

Table II. Ranges for $L_x(\gamma, \beta)$ graphs

Figure	β	γ	L_x
6	0.0 - 1.0	0.00 - 0.01	.001 - .007
7	0.0 - 1.0	0.01 - 0.10	.01 - .08
8	0.0 - 1.0	0.00 - 1.00	.01 - .10
9	0.0 - 1.0	0.20 - 1.00	.10 - .80

In order to use these charts, the parameters δ and β must be known. Since there is vast literature concerning this problem, we only make a few brief comments. There are at least three well-known methods to estimate parameters: maximum likelihood, graphical and linear. Maximum likelihood methods contain some computational and theoretical difficulties. The likelihood equations cannot be solved explicitly, and hence numerical algorithms must be used. For the two-parameter Weibull distribution, the order statistics are sufficient but not complete. Hence, small-sample optimality properties of the maximum-likelihood estimators can not be directly appealed to. Instead, asymptotic results must be used.

Graphical techniques are based on a graphical estimation of the cumulative distribution function based on the order statistics. The procedure consists of plotting failure times versus percent of sample that has failed on Weibull probability paper. Theoretically this data should lie on a straight line. Due to sampling errors, there will be some deviations from the theoretical straight line, and so a least square line is visually fit to the data points. The slope of this line provides an estimate of β, and the time point corresponding to 63% failures provides an estimate of δ. This graphical technique has been programmed on an APPLE computer graphics system by the authors. This provides one the advantage of being able to visually notice the degree of fit of the data to the model. In addition, an exact least squares line can be easily fit to the data. In order to use this program, the test engineer only needs to input the times to failure. The rest is done by the computer, including the estimates of the parameter. An example of this graphical technique on the APPLE computer system is provided in Figure 1 . The drawback of this method is that it assumes testing until all items fail, or at least a large percentage. Under Type I or II censoring, this procedure has to be modified.

Linear techniques are based on a few ordered statistics and are designed to provide best linear estimates under certain assumptions. They are easy to use and are adaptable to computer use by the maintaining of weights for the order statistics in a computer data base.

Full descriptions of these and other parameter estimation techniques are contained in [3].

7. Testing Procedure

The testing procedure suggested by the foregoing analysis is as follows:

(a) Determine the initial burn-in time, t_0.

(b) Based on the number of failures observed to time t_0, estimate δ and β. Calculate $\gamma = t_0/\delta$.

(c) Locate the values of γ and β on a chart and determine L_x and h. L_x is the expected proportional extra time that a failure-free test would run and h is the proportion of those systems still operating which will be expected to fail during that time.

(d) On the basis of L_x and h, make a decision to either

(i) Accept those components that are still operating, and re-test the failed components in a failure-free mode. This decision might be taken if h were very low.

(ii) Extend the test time on all systems to the new fixed time, $t_0(1 + L_x)$. At that time, accept those components still operating. This decision may be taken if the value of h is high, since by extending the test time there would be a high probability of finding more defectives.

Numerical Examples

(a) A set of n = 100 avionic systems is burned-in for an initial period of t_0 = 10 hours. At the end of this time period, 15 systems have failed and 85 are still operating. Based on the failure times of these components, estimates of β and δ are obtained as

$$\hat{\beta} = 0.56$$

$$\hat{\delta} = 320$$

An estimate of γ is then

$$\hat{\gamma} = t_0/\hat{\delta} = .031$$

From Figure A-3, L_x = .15 while from Figure B-2, h = .85. Hence,

(i) The expected length of a failure-free test is an additional 1.5 hours (10 x .15) per component.

(ii) If 1.5 hours of extra burn-in time are given equally to each of the systems still working, we would expect one additional new failure (85 x .01 = .85, rounded off to one). If this fixed-time procedure were done with replacement, then (using Equation 27), we would expect one repeat failure (.73, rounded off to one).

(iii) If the failure-free method were followed, we would expect (using Equation 26) an additional two repeat failures (2.05, rounded off to two).

(b) Under the same situation as above, the estimates of β and δ are:

$$\hat{\beta} = .32$$

$$\hat{\delta} = 66.7$$

An estimate of γ in this case is

$$\hat{\gamma} = t_0/\hat{\delta} = .15$$

From Figure A-4, the value of L_x is given as .7, while from Figure B-3, the value of h is given as .09. In this case,

(i) The expected length of a failure-free test is an additional 7 hours (10 x .7) per component.

(ii) If 7 hours of extra burn-in time are given equally to each of the systems still working, we would expect eight additional new failures (85 x .09, rounded off to eight). If this fixed-time procedure were done with replacement, then (using Equation 27) we would expect 6 repeat failures (5.8, rounded off to six).

(iii) If the failure-free method were followed, we would expect (using

Equation 26) an additional 30 repeat failures (30.44, rounded off to 30).

Since 9% of the working systems would thus fail in the next 7 hours, the decision may be made to extend the test an additional 7 hours and accept those systems still operating at the end of that time.

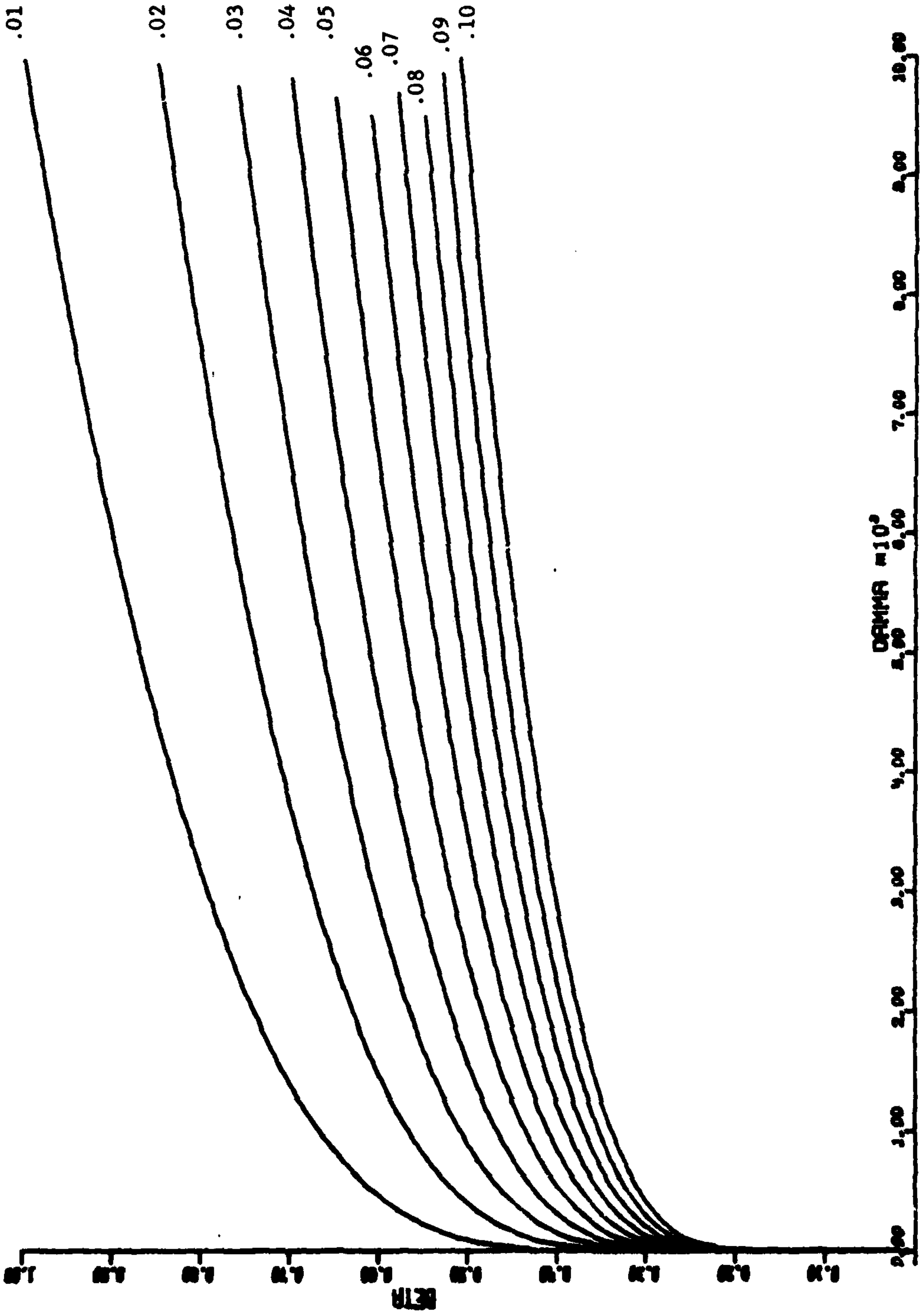
9. Conclusion and Recommendations

Based on these results, a decision on the use of a failure-free test should depend on the values of the parameters of the life-length distribution. Under certain conditions a failure-free test is desirable, while under other conditions it makes more sense to extend the test length evenly among all components. This report contains charts which provide the test engineer with the information necessary to make this decision.

An assumption underlying this report involves independence and identical distribution (iid) of the pre and post-repair lifetimes. On the other hand, this model allows for aging of components through the Weibull failure distribution. Models which assume an exponential failure distribution automatically imply iid pre and post-repair lifetimes due to the memoryless property of the exponential. Therefore, the Weibull assumption is a closer approximation than the exponential assumption. A possible extension of this research would be a relaxation of the iid assumption resulting in a better approximation to an actual reliability test.

APPENDIX A

Expected Test Length Charts



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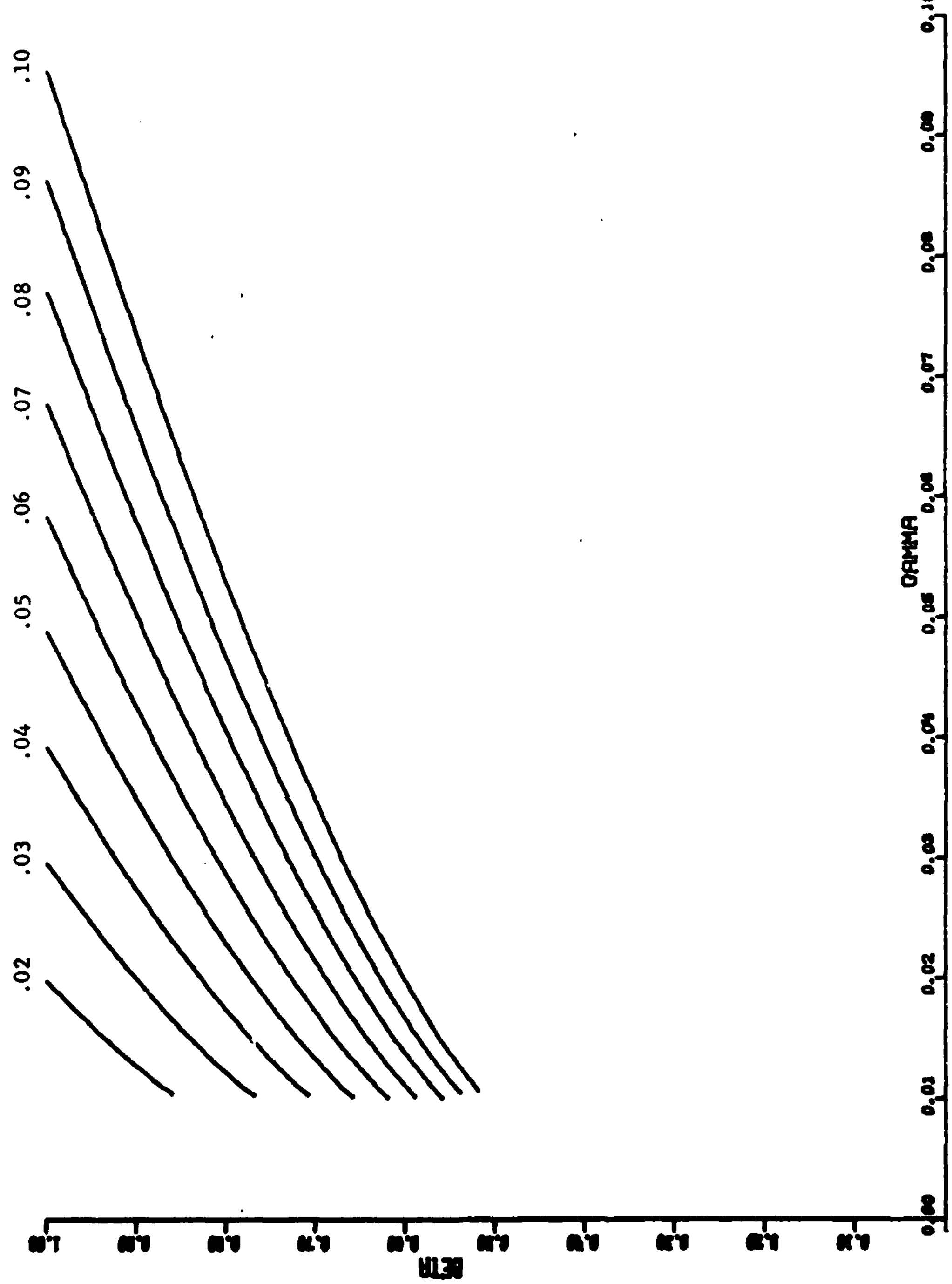

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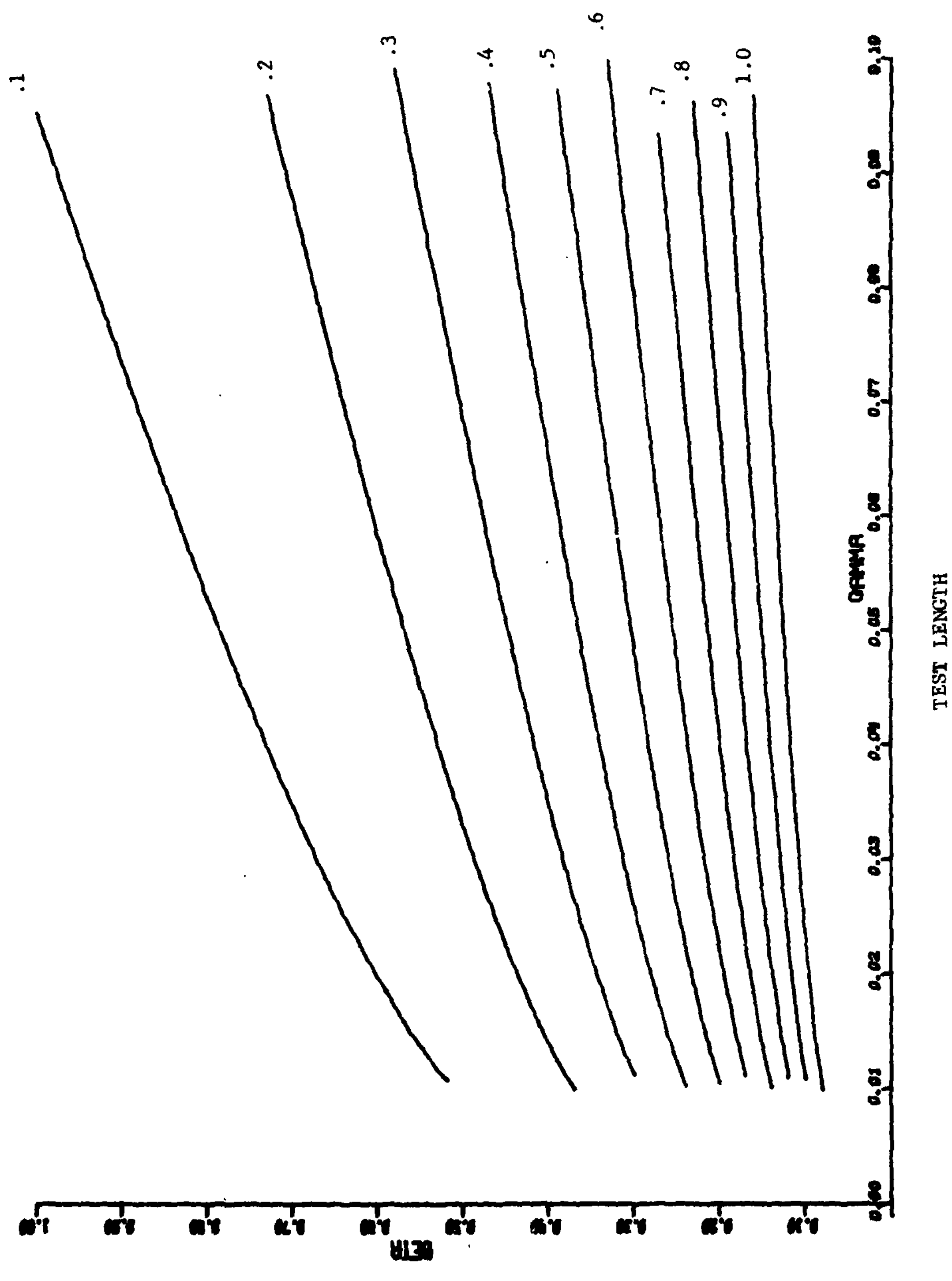
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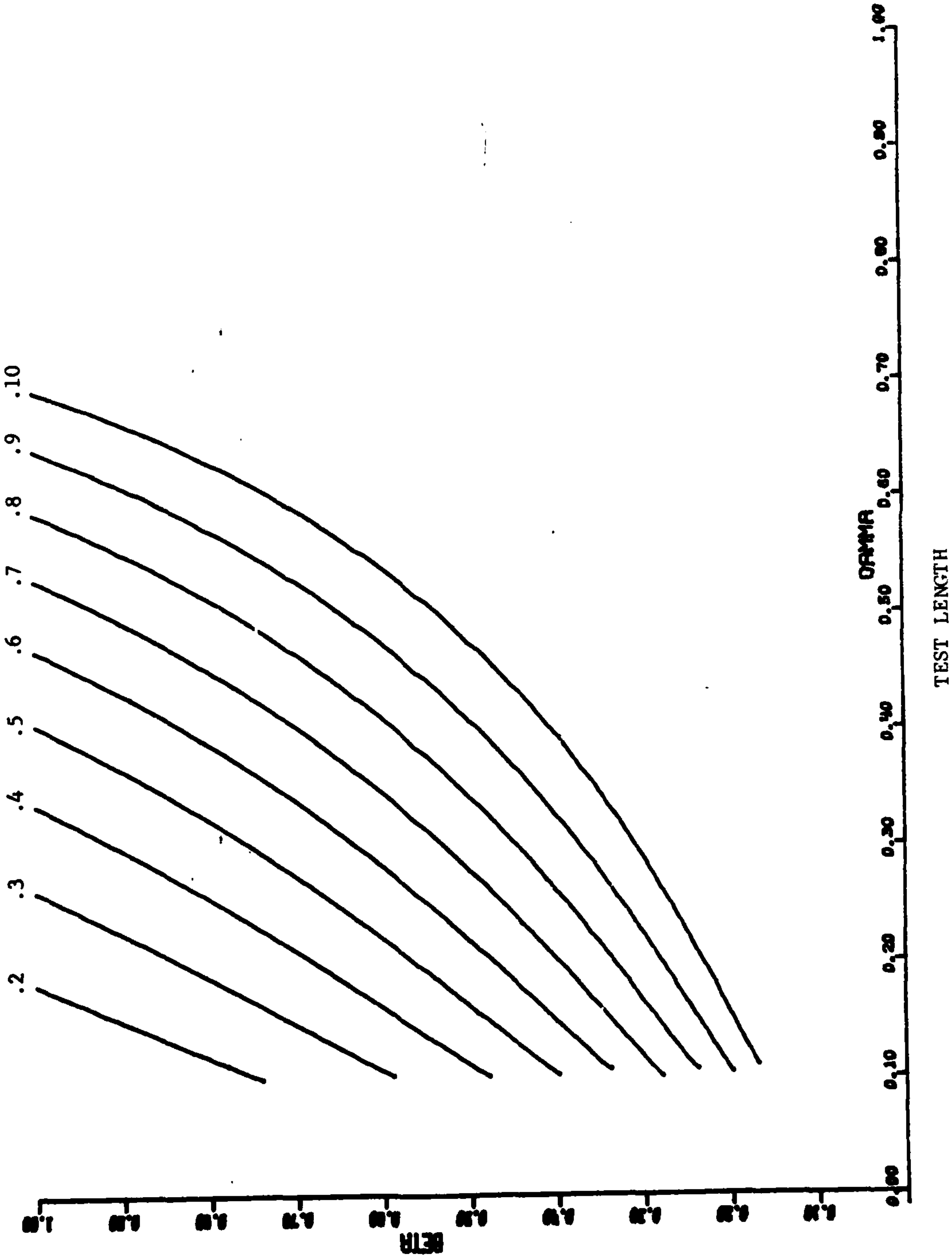

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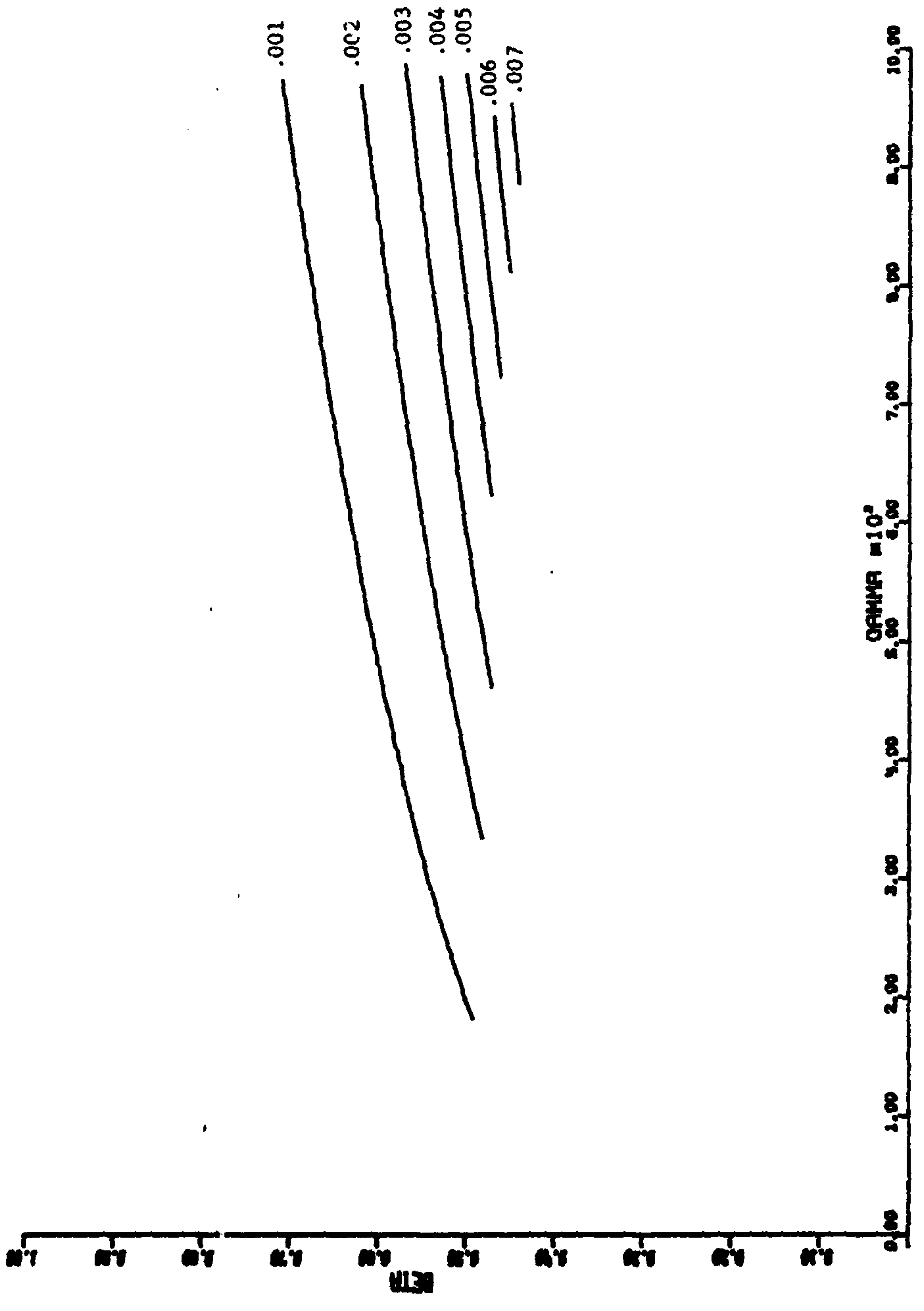
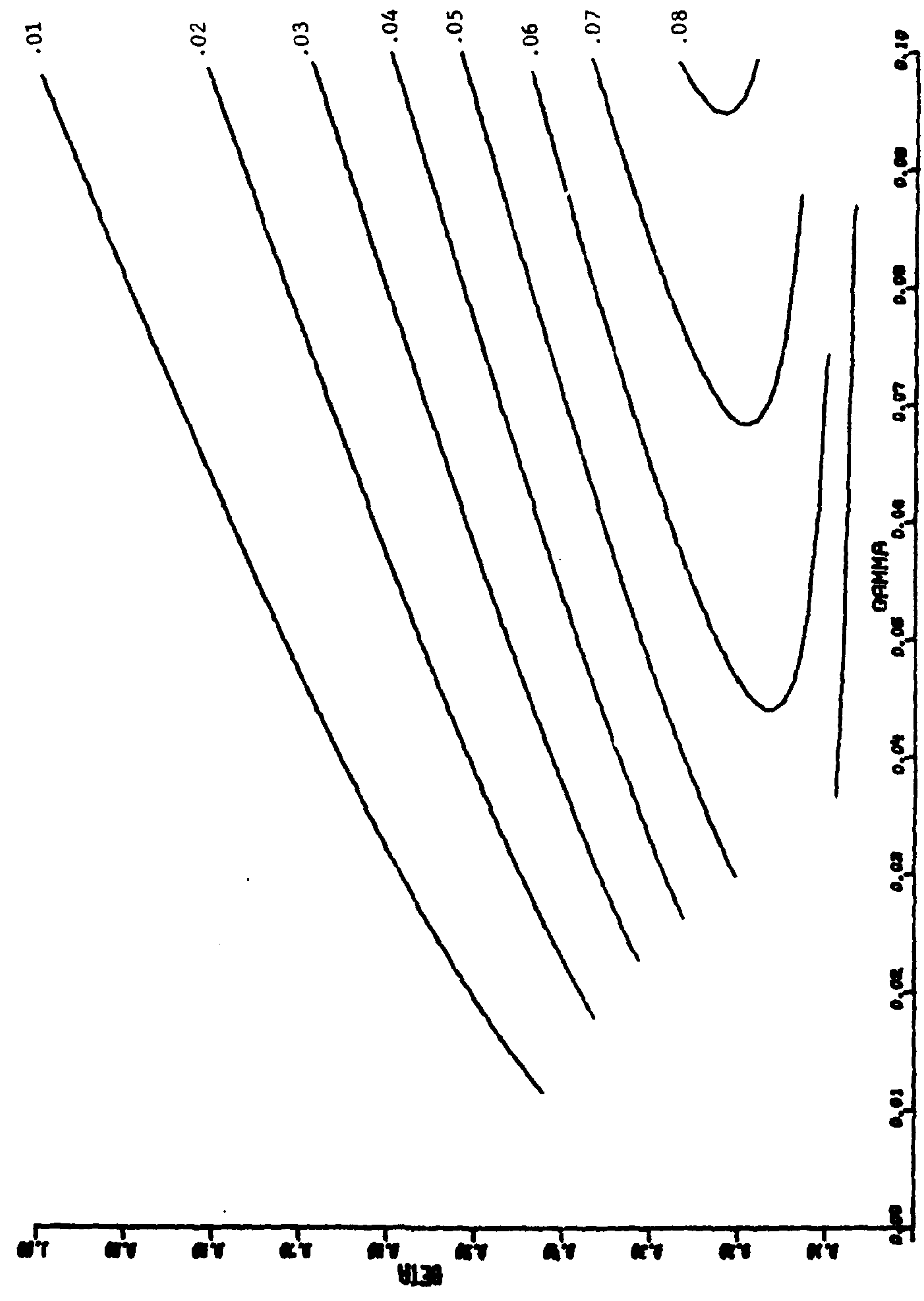
APPENDIX B

Conditional Failure Probability Charts



CONDITIONAL FAILURE PROBABILITY



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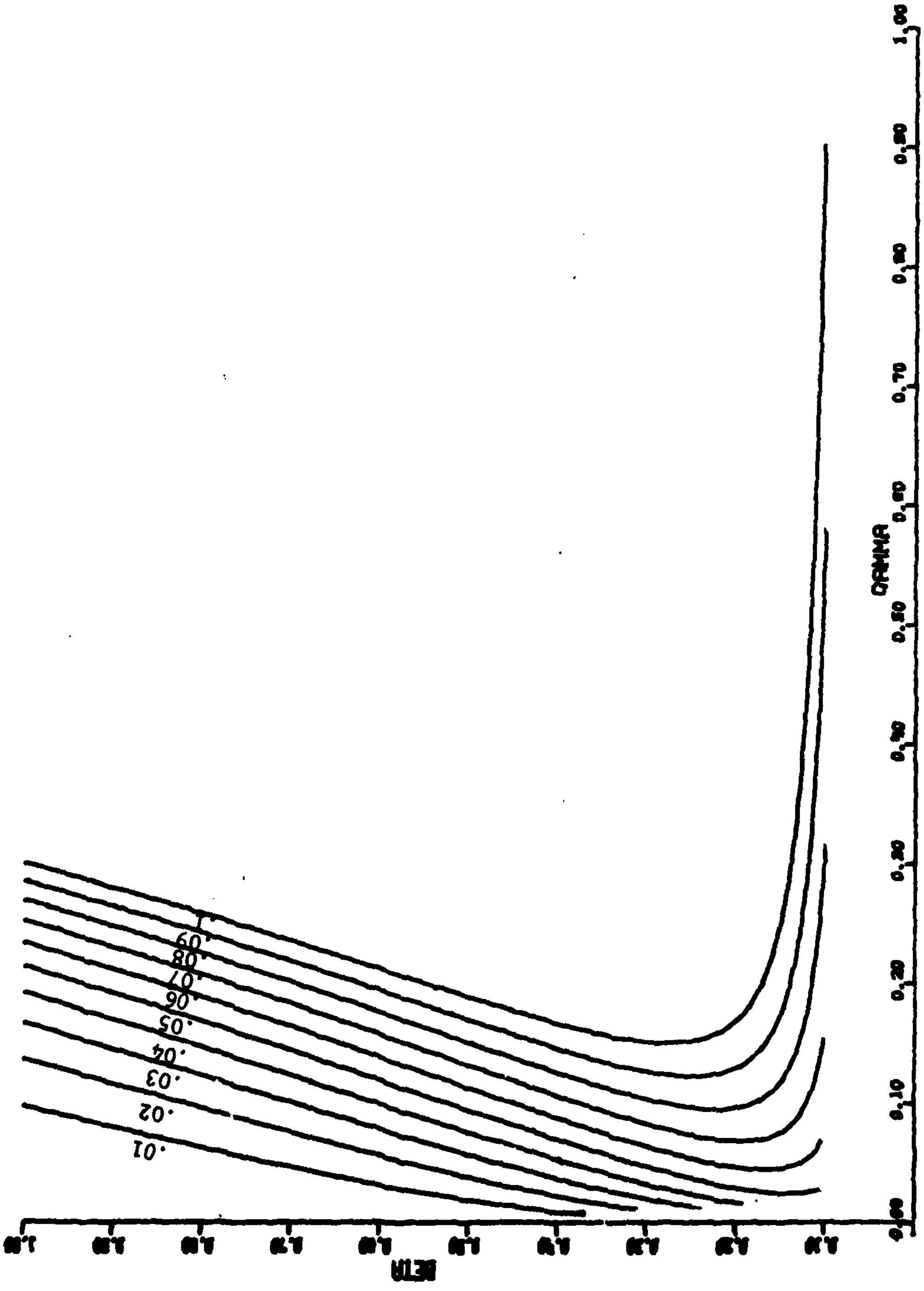

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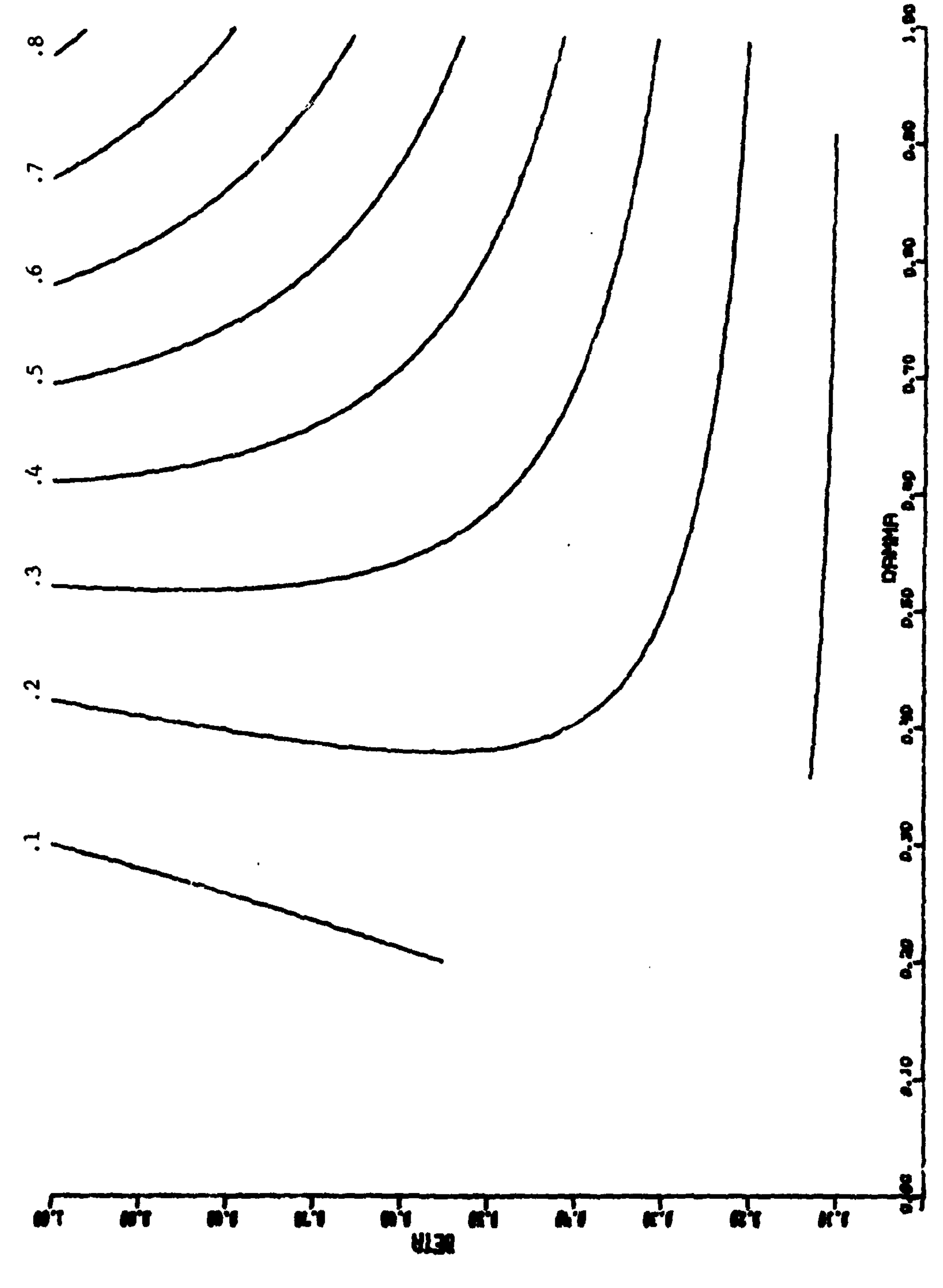

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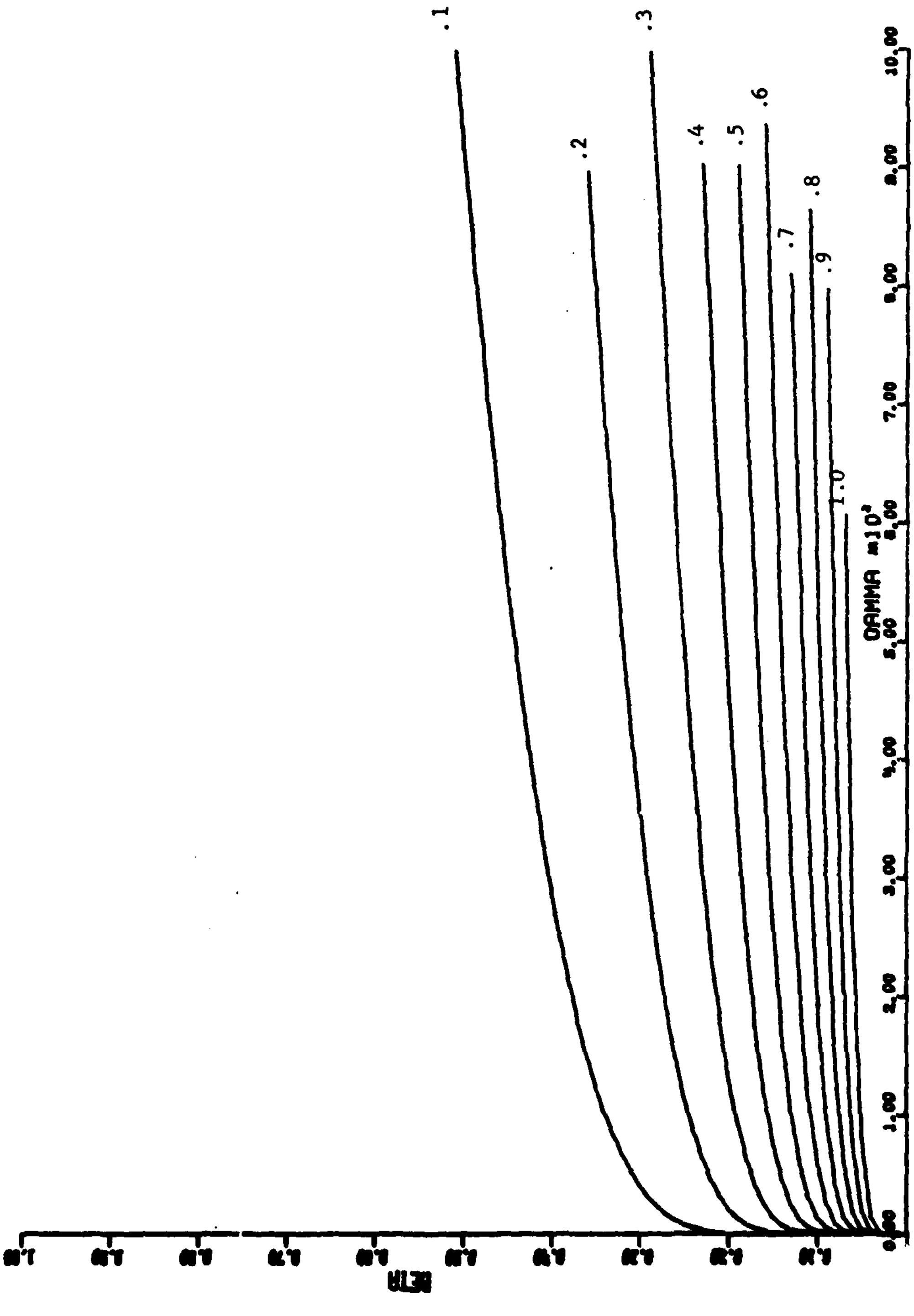
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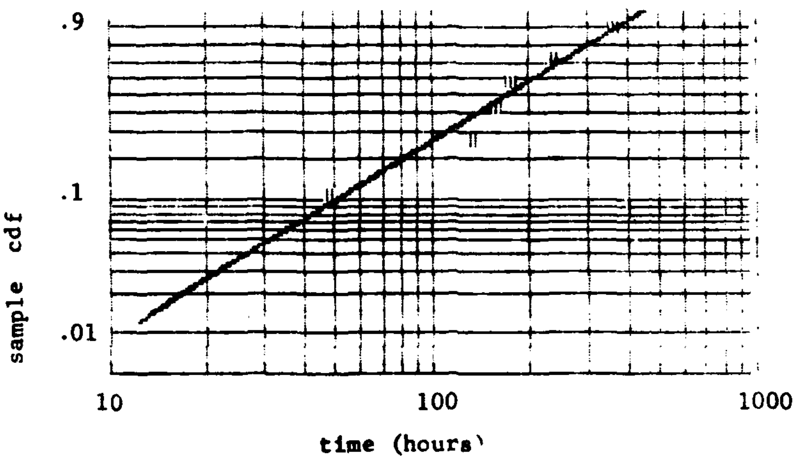

FAILURE TIMES

t_1-----------48 hours

t_2-----------132 hours

t_3-----------156 hours

t_4-----------175 hours

t_5-----------240 hours

t_6-----------360 hours

Figure 1. Estimation of parameters using graphical technique on the APPLE computer.

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